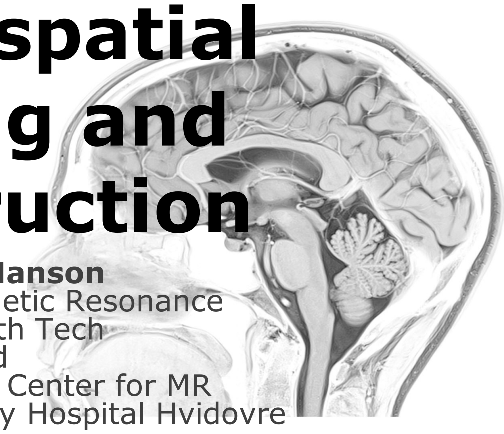
Echoes, spatial encoding and reconstruction



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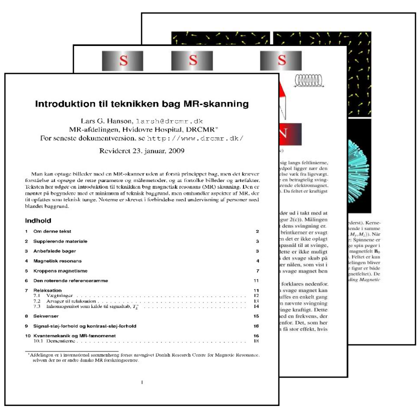
Center for Magnetic Resonance
DTU Health Tech Department of Health Technology

Danmarks Grundforskningsfond
Danish National Research Foundation

Danish Research Centre for Magnetic Resonance

Lecture notes

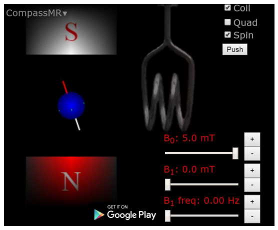
- Tutorial explaining basic MR and MRI in non-technical terms.

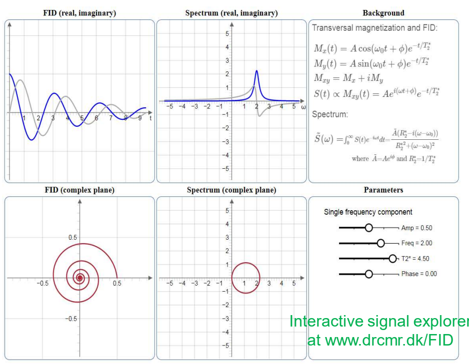

Introduktion til teknikken bag MR-skanning
Lars G. Hanson, larsh@drcmr.dk
MR-afdelingen, Hvidovre Hospital, DRCMR
For seneste dokumentversion, se http://www.drcmr.dk/
Revideret 23. januar 2009

Supplementary animations and software at http://drcmr.dk/MR

NMR and MRI simulators and animations

drcmr.dk/CompassMR

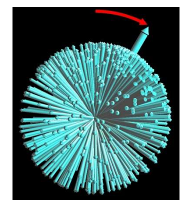




Interactive signal explorer
at www.drcmr.dk/FID

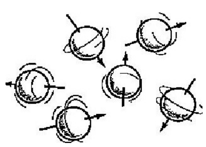
drcmr.dk/BlochSimulator

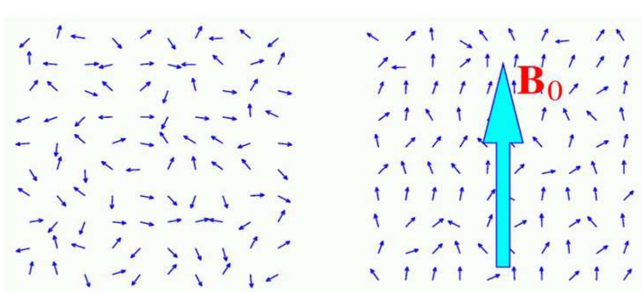
drcmr.dk/MR



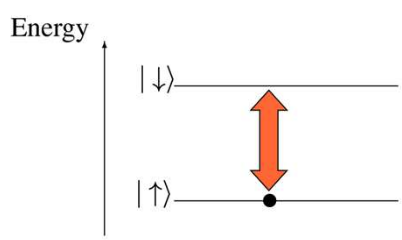
Magnetic Resonance

- Some atomic nuclei possess spin making them magnetic:





Spin-polarized sample

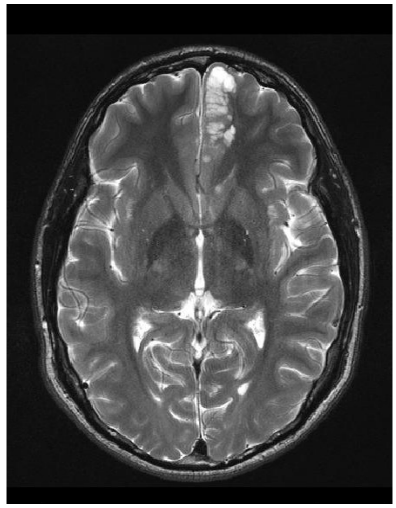


$$|\psi\rangle = a|S_z = \tfrac{1}{2}\rangle + b|S_z = -\tfrac{1}{2}\rangle$$

Same degrees of freedom, and same dynamics as classical magnetic dipoles.

Nuclear spins are rotated using magnetic fields (static and RF).

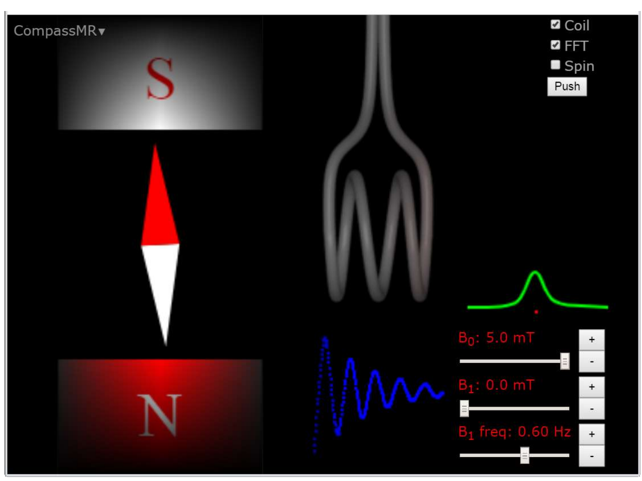
Typical MRI of an almost normal brain.



Intensity of a radio-wave signal!

How is the image recorded?

Compass Magnetic Resonance

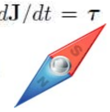


http://drcmr.dk/CompassMR or similar Android app

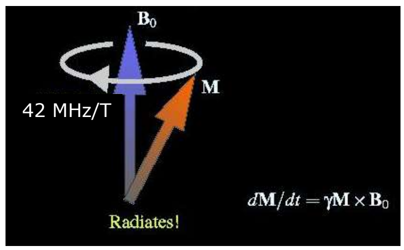
Classical equation of motion for nuclear spin

- The magnetic moment is proportional to the spin angular momentum: $\mu = \gamma \mathbf{J}$

- The angular momentum changes when a torque is applied: $d\mathbf{J}/dt = \tau$
- The magnetic field exerts a torque on the nuclei: $\frac{d\mathbf{J}}{dt} = \mu \times \mathbf{B}$

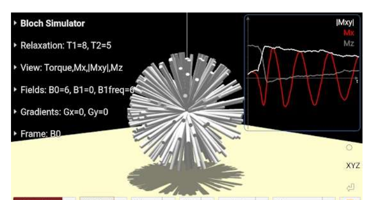


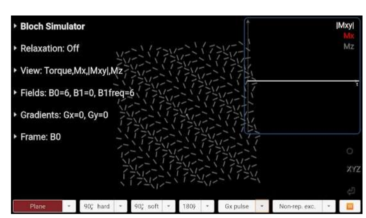
- The resulting equation of motion for a nuclear magnetic moment: $\frac{d\mu}{dt} = \gamma\mu \times \mathbf{B}$
- This describes precession of the magnetic moment around the magnetic field:



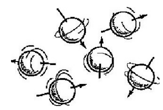
Bloch Simulator: http://drcmr.dk/Bloch

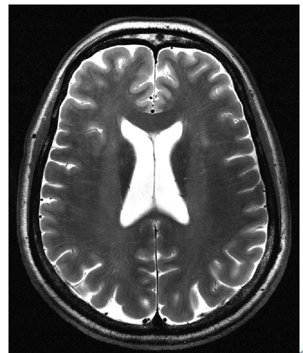
- Online simulator integrating Bloch equations:
  - shows nuclear spin dynamics, for various spin distributions, in precense of RF and gradient pulses
- Right menu shows parameters and choices.
- Bottom left button selects starting conditions
- Bottom row triggers events: RF and gradient pulses.
- Demonstration:
  - Excitation, precession around B0 and B1, rotating frames of reference, dephasing, echoes





Imaging ingredients





Signal generation:
- Oscillating magnetic fields matched to the nuclear oscillation frequency make them oscillate.
- The oscillation leads to reemission of radio waves. Recorded inductively.

– Radio wave properties.
  - Wavelength: Larger than coil dimensions!
    – Oscillating magnetic field, not "travelling waves"
  - Imaging problem:
    – These radio waves cannot be focused.

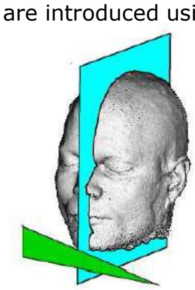
– Weird imaging causes weird artifacts!

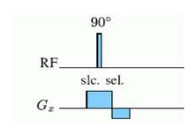
– Field gradients:
  - Linear main field variation in any direction.
    – made by gradient coils (electromagnets)
  - Makes the Larmor frequency vary across patient.

Online simulation: http://drcmr.dk/bloch

Gradients for slice selection

- Linear field variations are introduced using gradient coils:





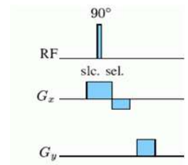
- Slice selection: Only nuclei on resonance are excited by a frequency-selective RF pulse.

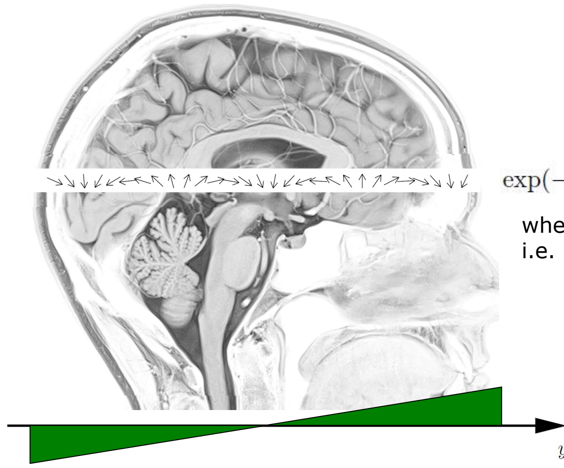
- The 3D imaging problem is reduced to a 2D problem.
  – In-plane resolution is needed.

Online simulation: http://drcmr.dk/bloch

In-plane gradients cause phase rolls

- Gradient applied in-plane after slice selection generate Mxy phase roll:





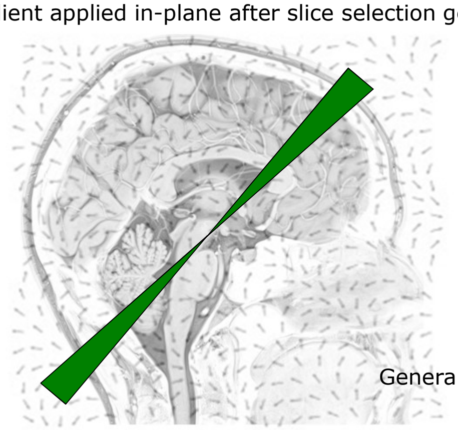
$\exp(-i2\pi k_y y)$

where $k_y = 1/\lambda_y$
i.e. inverse wavelength

Online simulation: http://drcmr.dk/bloch

In-plane gradients cause phase rolls

- There are not only nuclei on a line from neck to nose: 2D-distribution.
- Any gradient applied in-plane after slice selection generates a phase roll:

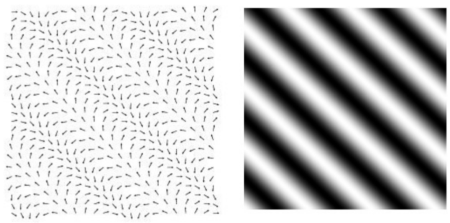


General phase roll: $\exp(-i2\pi \mathbf{k} \cdot \mathbf{r})$

A sum of gradients is a gradient that also cause a linear phase roll.

The Fourier theorem: Every image can be expressed as a sum of phase-rolls

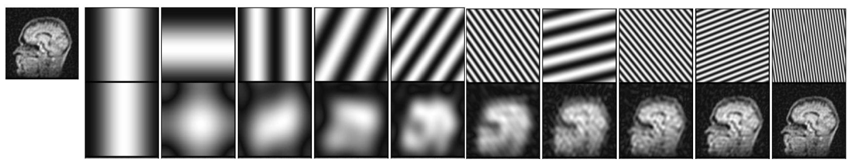
- Color-coding phase-rolls for alternative visulization:



$$\exp(-i2\pi\mathbf{k}\cdot\mathbf{r})$$

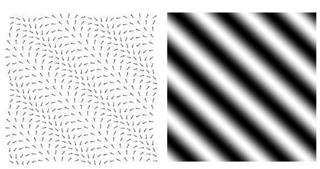
Teaser: Any image can be perceived as a sum of stripe patterns and measuring the “stripedness” is enough to do imaging.

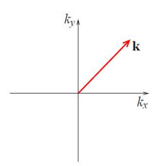
- The Fourier theorem: Any distribution of transversel magnetization can be perceived as a weighted sum of phase rolls with weights S(**k**).



Measuring S(k) by refocusing phase rolls

- Book-keeping of phase rolls: One phase roll for every position in k-space

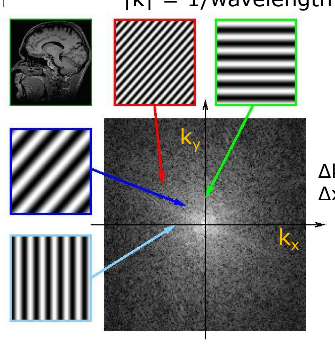




k-direction indicates direction of variation of the phase roll.

Length indicate stripe density: |k| = 1/wavelength

- The structure of k-space:
  - Math: $M_{xy}(\mathbf{r})$ can be perceived as a weighted sum of phase rolls.
  - Refocus a roll to measure weight
    - Spatial integration by coil (Fourier integral)
  - k-space is infinite.
    - Central part is most important

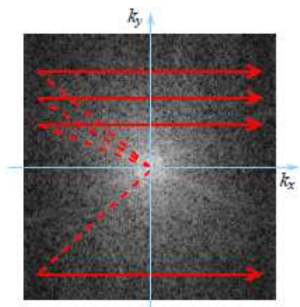


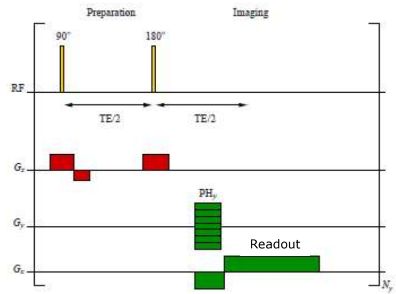
$\Delta k = 1/FOV$
$\Delta x = 1/(2k_{x,max})$

Detail: An unimportant axis change was made compared to earlier slides.

Traversing k-space

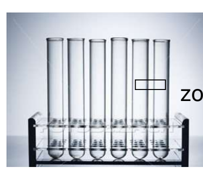
- The signal is measured during refocusing of each and every phase-roll.
- Phase rolls change as long as gradients are applied.
  - the gradient **G** is the velocity in k-space: $\mathbf{G} = \gamma^{-1} d\mathbf{k}/dt$
  - the signal can be measured while the phase roll changes
  - 180° pulses mirror phase rolls: **k** becomes -**k**





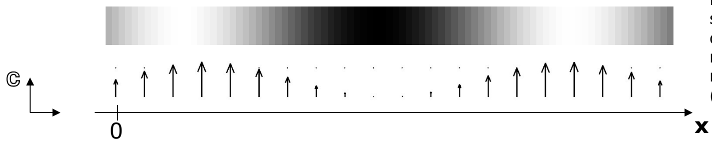
k-space and imaging

- Imaging is here exemplified. The k-space representation is provided explicitly for a striped object: A row of water filled test tubes put in the scanner.



- The following slides show how imaging works with the object being the content of the zoom box above (water excited by a 90 degree pulse).
- The graphs and decomposition will essentially remain unchanged for a larger part of the ”striped” object (the zoom is chosen to keep graphs readable).
- The formation of phase rolls can be explored with the Bloch Simulator, http://drcmr.dk./bloch. See also YouTube https://www.youtube.com/watch?v=qXhQhgvpRU0 for spin and signal evolution during imaging of a similar object.

Object with variation in the transversal magnetization after excitation e.g due to varying water content or relaxation:

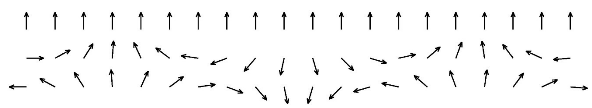


Note: The gray-scale stripe pattern here does not signify a phase roll, but transversal magn. density variation (unlike in other graphs)

The Fourier theorem: Any complex function ($M^o(x)$) can be written as a sum of complex phase roll patters. Which ones in this case?

1 *
+ 0.5*
+ 0.5*

=



i*exp(0x)
exp(+ikx)
-exp(-ikx)

So we only get significant signal at 3 positions in k-space:
S(0) = i, S(k)=0.5, S(-k)=-0.5

Only proportionality!

Image reconstruction:
$M^0(x)$ = i*exp(0x)+0.5*(exp(+ikx)-exp(-ikx)) = i(1+sin(kx))

Take home messages

- MR measurements involve RF and gradient pulses.
  - On-resonance RF pulses do rotations around a transversal axis.
    - This can lead to later refocusing, for example (spin echoes)
  - Gradients
    - turn spatial axis into frequency axis
    - introduce and remove phase rolls.
- Imaging in a nut shell:
  - An MR image is showing the distribution of transversal magnetization.
  - Any such distribution can be perceived as a weighted sum of phase rolls.
  - Phase rolls do not give signal until they are refocused.
  - When refocused, the signal strength is the “stripedness” of the patient.
  - “Image reconstruction” is adding stripe patterns weighted by the measured stripedness.

Thank you for your attention.