Diffusion MRI acquisition basics

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Problems part A: From gradient waveform to b-value

1. What is the b-value for the waveform in Figure 1 (right side)? How large will the b-value become if all gradient axes play the same waveform simultaneously? Assume the gyromagnetic ratio $\gamma = 2.68 \cdot 10^8$ $(\mathrm{s} \cdot \mathrm{T})^{-1}$ and $\gamma^2 = 7.16 \cdot 10^8$ $(\mathrm{s} \cdot \mathrm{T})^{-2}$.

2. Starting from the general definition of the b-value for linear encoding

$$b = \int_0^\tau q^2(t)\mathrm{d}t \text{ where } q(t) = \int_0^t g(t')\mathrm{d}t', \tag{1}$$

   where $g(t)$ is the gradient of the Larmor frequency, derive a simplified expression of the b-value for rectangular gradient pulses as shown in Figure 1 (ignore ramp time). What is the diffusion time? Hint: It is not always $\Delta - \delta/3$!

3. **Bonus question:** Sketch the q-trajectory for a sequence where each gradient pulse is replaced by a bipolar pulse. Assuming a spin echo sequence, discuss what must be added to the sequence diagram if we use such bipolar pulses on either side of the refocusing pulse.

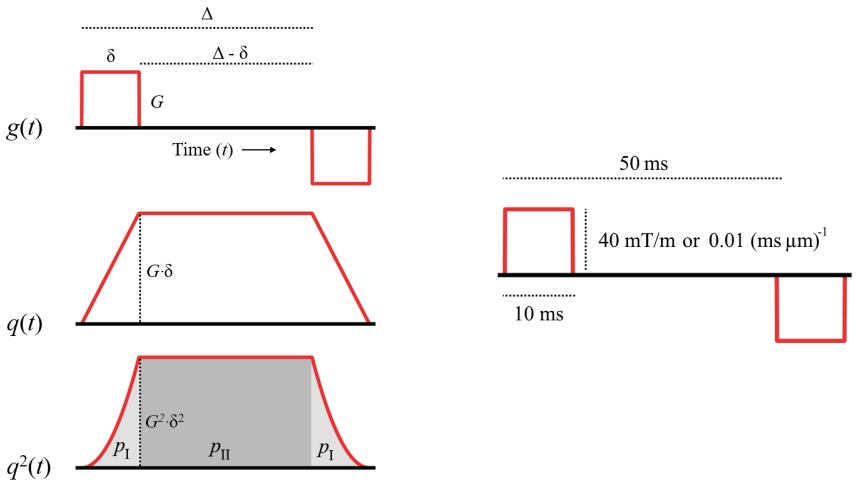

Figure 1: (left) Gradient and dephasing vectors, g and q, sketched as a function of time to support the derivation of a simplified expression of b in problem 1.1. (right) Example waveform for problem 1.2, where a single axis (red) plays two trapezoidal pulses of field gradient amplitude 40 mT/m or Larmor frequency gradient of approximately 0.01 $(\mathrm{ms} \cdot \mu\mathrm{m})^{-1}$, duration 10 ms and gradient onsets separated by 50 ms.

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Problems part B: Imaging and diffusion encoding

Let's start by sketching one shot of the PGSE sequence including diffusion preparation and read-out.

1. The imaging parameters and hardware specifications define the achievable b-value echo time combinations and thus the space available for optimization. For a given acquisition with fixed imaging parameters and given hardware limits, work out the maximal b-value for a TE of 80ms. The imaging parameters are a resolution of 2mm isotropic, FOV 200x200mm and no in-plane acceleration is used. The hardware specifications are $Gmax = 40\frac{mT}{m}$ or $0.01(\text{ms}\mu\text{m})^{-1}$, gyromagnetic ratio $\gamma^2 = 7.2 \cdot 10^{16}(s^2T^2)^{-1}$. (Scanner A).
   Assumptions: The inter-echo-spacing is given as 0.5ms, crushers, slice rewinder, read rewinder and phase rewinder gradients as well as all pulses are instantaneous events (duration = 0). The slew rate is infinite for the following calculations.

2. We need higher b-values! What are possible solutions? What will be the effects of these 5 possible solutions. (Assumptions including inter-echo spacing and TE=80ms remain)

   (a) We buy scanner B with Gmax = 80mT/m $(0.02(\text{ms}\mu\text{m})^{-1})$.

   (b) We add in-plane acceleration using parallel imaging (SENSE/GRAPPA) with factor 2.

   (c) We add in-plane acceleration using partial Fourier/half scan with factor 3/5.

   (d) We reduce the resolution from 2mm to 4mm isotropic

   (e) We add simultaneous-multi-slice (SMS) imaging (factor 3).

3. We have settled on a slice thickness 2mm and a brain coverage of 100mm. The full shot duration including fat saturation equals 150ms. We have decided on three repetitions of $b = 0\frac{\text{ms}}{\mu\text{m}^2}$ and 32 directions on our $b = 3\frac{\text{ms}}{\mu\text{m}^2}$ shell. What is the minimal total acquisition time assuming no SMS and SMS factor 2?

4. **Bonus questions:**

   (a) Is there potential for unwanted interaction between the imaging gradients and the diffusion preparation? Under what conditions are these important?

   (b) Having worked out all these timings and given the restriction imposed by the time of the second diffusion gradient - it seems like a great idea to fill up the time between the first diffusion gradients and the refocusing pulse! Is this a good idea? What other changes would be required?

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Answers to pt A

1. The numbers from Figure 1 can be plugged into the equation above in terms of the gradient of the Larmour frequency (in $(\text{ms}\cdot\mu\text{m})^{-1}$) and times (in ms)

$$b = (0.0107 \cdot 10)^2 \cdot (50 - 10/3) = 0.53 \text{ ms}/\mu\text{m}^2 \tag{2}$$

or, equally correct but slightly more cumbersome, in terms of the field gradient and time in non-scaled units

$$b = (2.68 \cdot 10^8 \cdot 40 \cdot 10^{-3} \cdot 10 \cdot 10^{-3})^2 \cdot (50 \cdot 10^{-3} - 10 \cdot 10^{-3}/3) = 0.53 \text{ ms}/\mu\text{m}^2 \tag{3}$$

which is equal to $5.3 \cdot 10^8$ s/m^2 and 530 s/mm^2. If all three axes play this waveform at the same time, the gradient amplitude becomes

$$G' = \sqrt{G^2 + G^2 + G^2} = G \cdot \sqrt{3} \tag{4}$$

and since $b \propto G^2$ it increase by a factor of three, i.e., $b' = b \cdot 3 \approx 1.6$ ms/μm^2.

2. The b-value is the integral of $q^2(t)$, and we can separate that trajectory into three parts ($b = p_\text{I} + p_\text{II} + p_\text{III}$), as seen in Figure 1. The first, and last parts have the same integral ($p_\text{I} = p_\text{III}$), according to

$$p_\text{I} = \int_0^\delta G^2 t^2 \text{d}t = G^2 \left[\frac{t^3}{3}\right]_0^\delta = \frac{G^2\delta^3}{3} \tag{5}$$

where G is the amplitude of the larmour frequency gradient. The second part is simply the area of a rectangle with height $G^2\delta^2$ and width $(\Delta - \delta)$, so that

$$p_\text{II} = G^2\delta^2(\Delta - \delta). \tag{6}$$

The b-value is therefore

$$b = 2\frac{G^2\delta^3}{3} + G^2\delta^2(\Delta - \delta) = G^2\delta^2\left(\Delta - \frac{\delta}{3}\right). \tag{7}$$

The diffusion time may not have a straightforward definition based on the experiment since it depends on the tissue architecture. This means that any experiments aimed at probing time-dependent diffusion should report the entire shape of the diffusion encoding waveform to be fully interpretable, e.g. Δ and δ for the Stejskal-Tanner design (see van Gelderen et al. JMR B 1994; Novikov et al. NMR Biomed 2018).

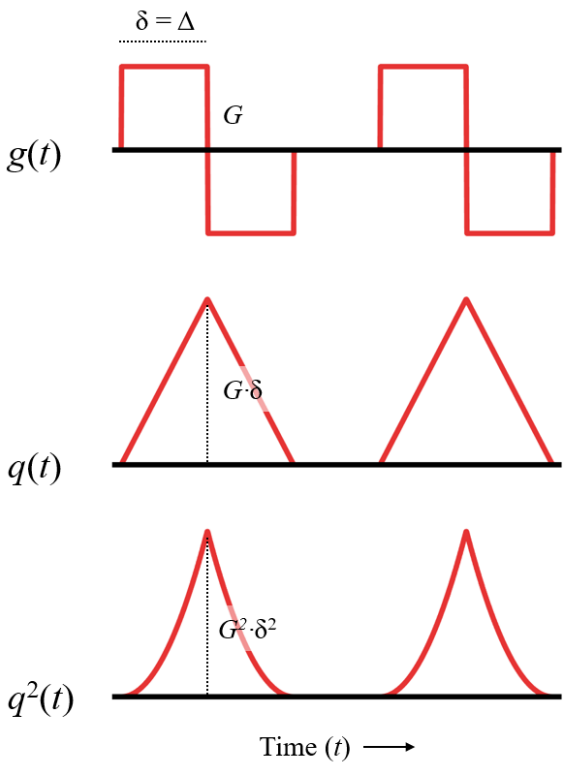

Figure 2: Sketch of sequence where monopolar gradients are replaced by a bipolar configuration.

3. The trajectory of a pair of bipolar pulses is shown in Figure 2. The efficiency is reduced since the q-vector is zero for a large part of the experiment, and therefore contributes no diffusion encoding. The sequence must include crusher gradients since the diffusion encoding no longer has a non-zero gradient moment after the refocusing pulse.

Answers to pt B

See Figure 3.

1. **First, we need to work out the length of the read-out:** With a FOV of 200x200mm and a resolution of 2x2mm, the matrix size is 100x100. Given that no in-plane acceleration is used, we thus need 100 phase encoding steps. The total length is defined by the inter-echo spacing of 0.5ms $\cdot 100 = 50$ms.
   Now, lets work out the times available for the diffusion preparation: This sets the time available for the diffusion preparation. The time available for the second diffusion gradient equals $80 - 40 - 25 = 15$ms and thus defines the maximal $\delta = 15$ms. Finally, the Δ is obtained from the time between the pulses (earliest start of the diffusion gradients), resulting here in $\Delta = 40$ms.

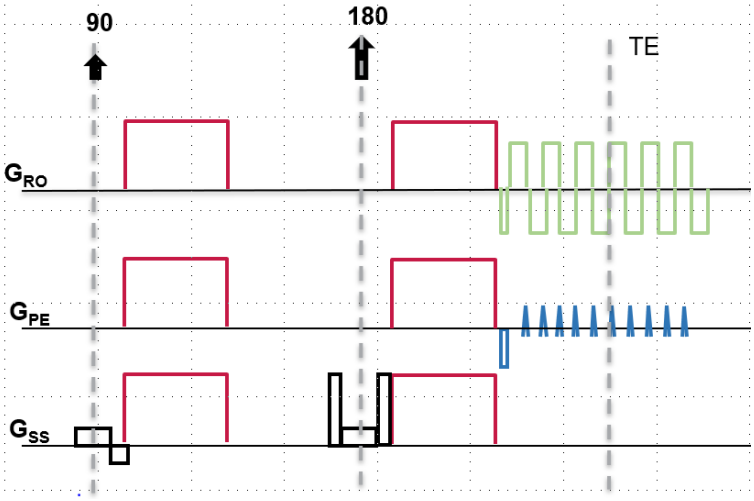

Figure 3: PGSE sequence with EPI-readout

Finally, the b-value:
Using the formula derived in Problem 1, the maximal achievable b-value under idealistic scenarios equals $b = G^2 \cdot \delta^2(\Delta - \frac{\delta}{3}) = 0.907 \frac{\text{ms}}{\mu\text{m}^2}$.

2. (a) The higher Gmax affects only step 3 - resulting thus in $b = 3.628 \frac{\text{ms}}{\mu\text{m}^2}$.

   (b) In-plane acceleration by parallel imaging (factor 2) reduces the total time for the read-out by 2 to $50/2 = 25ms$, this increases the time available between refocusing pulse and begin of the read-out to $80 - 40 - 25/2ms = 27.5$ms. Finally, a higher b-value of $2.686 \frac{\text{ms}}{\mu\text{m}^2}$ can be achieved.

   (c) Half-scan acceleration 3/5 (0.6) reduces the length of the read-out by 2/5 to 30[ms], however, this has a bigger effect on the time between refocusing and read-out begin - resulting in $80 - 40 - 5 = 35$ms, finally giving a b-value of $4 \frac{\text{ms}}{\mu\text{m}^2}$.

   (d) The reduction of the resolution reduces the required phase-encoding steps to 200mm/4mm=50 steps - thus halving time to 50*0.5ms=25ms, increasing available time to $80 - 40 - 25/2 = 27.5$ms (similar to SENSE 2) thus equally resulting in a maximal b-value of $2.686 \frac{\text{ms}}{\mu\text{m}^2}$.

   (e) SMS does not change the timings here (at least not in idealized scenarios where the pulses are assumed to be instantaneous). However, in reality SMS pulses tend to be longer, thus reducing available time for diffusion weighting. The benefit of SMS lies in reducing the number of shots required and thus the TR.

3. The minimal theoretical TR depends on the number of slices and the shot length. We need 100/2mm=50 slices, resulting with the given shot length in a TR of 7.5s. Taking the 32+3 volumes, the total time is $35 \cdot 7.5s = 4$min32sec. The difference between this

theoretical and the practical TR as well as the non-linear decay is due to the gradient duty cycle.

4. (a) All gradients, including imaging gradients and crusher gradients are contributing to the diffusion weighting. Extreme cases include very low b-values as well as cases where the crushers are uniquely on one axis (often on the z-axis).

   (b) If the first diffusion gradient is "filled up", there needs to be a change in polarity to assure the 0th moment remains 0. Further effects to be taken into account are e.g. Maxwell terms and eddy currents.